

10.5 | RC Circuits

Learning Objectives

By the end of the section, you will be able to:

- Describe the charging process of a capacitor
- Describe the discharging process of a capacitor
- List some applications of RC circuits

When you use a flash camera, it takes a few seconds to charge the capacitor that powers the flash. The light flash discharges the capacitor in a tiny fraction of a second. Why does charging take longer than discharging? This question and several other phenomena that involve charging and discharging capacitors are discussed in this module.

Circuits with Resistance and Capacitance

An **RC circuit** is a circuit containing resistance and capacitance. As presented in **Capacitance**, the capacitor is an electrical component that stores electric charge, storing energy in an electric field.

Figure 10.38(a) shows a simple RC circuit that employs a dc (direct current) voltage source \mathcal{E} , a resistor R , a capacitor C , and a two-position switch. The circuit allows the capacitor to be charged or discharged, depending on the position of the switch. When the switch is moved to position A, the capacitor charges, resulting in the circuit in part (b). When the switch is moved to position B, the capacitor discharges through the resistor.

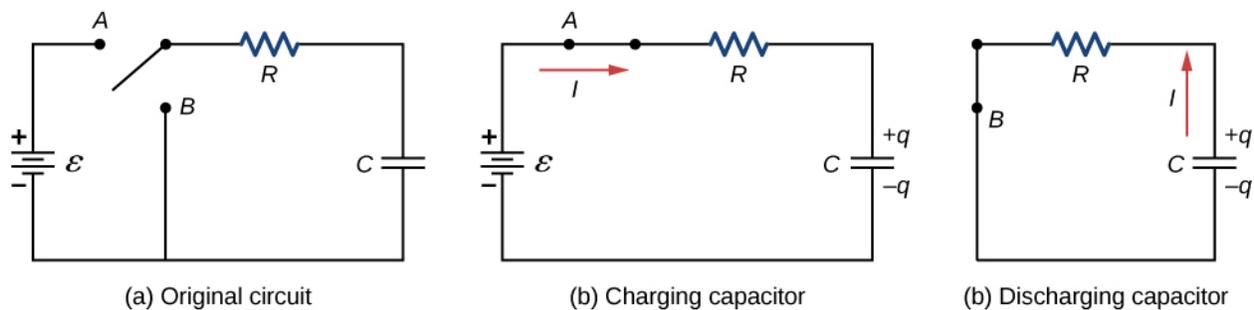


Figure 10.38 (a) An RC circuit with a two-pole switch that can be used to charge and discharge a capacitor. (b) When the switch is moved to position A, the circuit reduces to a simple series connection of the voltage source, the resistor, the capacitor, and the switch. (c) When the switch is moved to position B, the circuit reduces to a simple series connection of the resistor, the capacitor, and the switch. The voltage source is removed from the circuit.

Charging a Capacitor

We can use Kirchhoff's loop rule to understand the charging of the capacitor. This results in the equation $\mathcal{E} - V_R - V_C = 0$.

This equation can be used to model the charge as a function of time as the capacitor charges. Capacitance is defined as $C = q/V$, so the voltage across the capacitor is $V_C = \frac{q}{C}$. Using Ohm's law, the potential drop across the resistor is

$V_R = IR$, and the current is defined as $I = dq/dt$.

$$\begin{aligned}\mathcal{E} - V_R - V_C &= 0, \\ \mathcal{E} - IR - \frac{q}{C} &= 0, \\ \mathcal{E} - R\frac{dq}{dt} - \frac{q}{C} &= 0.\end{aligned}$$

This differential equation can be integrated to find an equation for the charge on the capacitor as a function of time.

$$\varepsilon - R \frac{dq}{dt} - \frac{q}{C} = 0,$$

$$\frac{dq}{dt} = \frac{\varepsilon C - q}{RC},$$

$$\int_0^q \frac{dq}{\varepsilon C - q} = \frac{1}{RC} \int_0^t dt.$$

Let $u = \varepsilon C - q$, then $du = -dq$. The result is

$$-\int_0^q \frac{du}{u} = \frac{1}{RC} \int_0^t dt,$$

$$\ln\left(\frac{\varepsilon C - q}{\varepsilon C}\right) = -\frac{1}{RC}t,$$

$$\frac{\varepsilon C - q}{\varepsilon C} = e^{-\frac{t}{RC}}.$$

Simplifying results in an equation for the charge on the charging capacitor as a function of time:

$$q(t) = C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right) = Q\left(1 - e^{-\frac{t}{\tau}}\right). \quad (10.8)$$

A graph of the charge on the capacitor versus time is shown in **Figure 10.39(a)**. First note that as time approaches infinity, the exponential goes to zero, so the charge approaches the maximum charge $Q = C\varepsilon$ and has units of coulombs. The units of RC are seconds, units of time. This quantity is known as the time constant:

$$\tau = RC. \quad (10.9)$$

At time $t = \tau = RC$, the charge is equal to $1 - e^{-1} = 1 - 0.368 = 0.632$ of the maximum charge $Q = C\varepsilon$. Notice that the time rate change of the charge is the slope at a point of the charge versus time plot. The slope of the graph is large at time $t = 0.0$ s and approaches zero as time increases.

As the charge on the capacitor increases, the current through the resistor decreases, as shown in **Figure 10.39(b)**. The current through the resistor can be found by taking the time derivative of the charge.

$$I(t) = \frac{dq}{dt} = \frac{d}{dt}\left[C\varepsilon\left(1 - e^{-\frac{t}{RC}}\right)\right],$$

$$I(t) = C\varepsilon\left(\frac{1}{RC}\right)e^{-\frac{t}{RC}} = \frac{\varepsilon}{R}e^{-\frac{t}{RC}} = I_0 e^{-\frac{t}{RC}},$$

$$I(t) = I_0 e^{-t/\tau}. \quad (10.10)$$

At time $t = 0.00$ s, the current through the resistor is $I_0 = \frac{\mathcal{E}}{R}$. As time approaches infinity, the current approaches zero.

At time $t = \tau$, the current through the resistor is $I(t = \tau) = I_0 e^{-1} = 0.368I_0$.

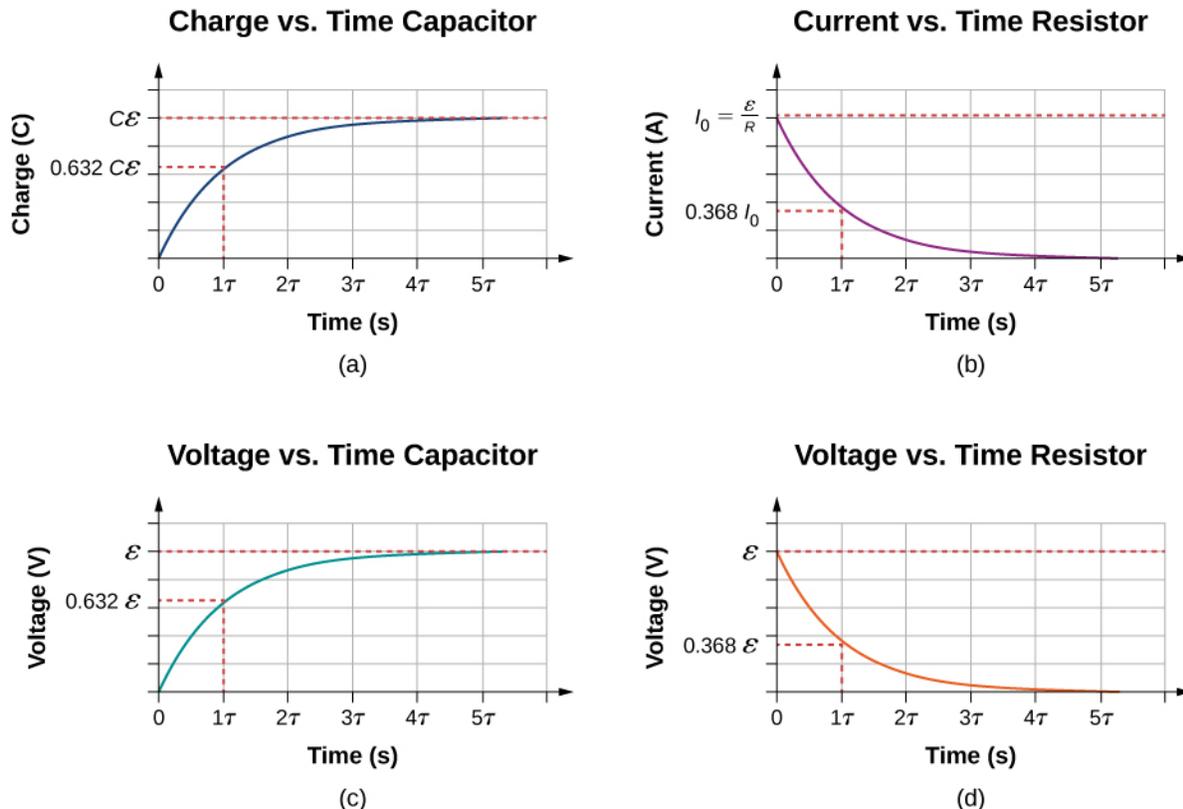


Figure 10.39 (a) Charge on the capacitor versus time as the capacitor charges. (b) Current through the resistor versus time. (c) Voltage difference across the capacitor. (d) Voltage difference across the resistor.

Figure 10.39(c) and **Figure 10.39(d)** show the voltage differences across the capacitor and the resistor, respectively. As the charge on the capacitor increases, the current decreases, as does the voltage difference across the resistor $V_R(t) = (I_0 R)e^{-t/\tau} = \mathcal{E}e^{-t/\tau}$. The voltage difference across the capacitor increases as $V_C(t) = \mathcal{E}(1 - e^{-t/\tau})$.

Discharging a Capacitor

When the switch in **Figure 10.38(a)** is moved to position *B*, the circuit reduces to the circuit in part (c), and the charged capacitor is allowed to discharge through the resistor. A graph of the charge on the capacitor as a function of time is shown in **Figure 10.40(a)**. Using Kirchhoff's loop rule to analyze the circuit as the capacitor discharges results in the equation $-V_R - V_C = 0$, which simplifies to $IR + \frac{q}{C} = 0$. Using the definition of current $\frac{dq}{dt}R = -\frac{q}{C}$ and integrating the loop equation yields an equation for the charge on the capacitor as a function of time:

$$q(t) = Qe^{-t/\tau}. \quad (10.11)$$

Here, Q is the initial charge on the capacitor and $\tau = RC$ is the time constant of the circuit. As shown in the graph, the charge decreases exponentially from the initial charge, approaching zero as time approaches infinity.

The current as a function of time can be found by taking the time derivative of the charge:

$$I(t) = -\frac{Q}{RC}e^{-t/\tau}. \quad (10.12)$$

The negative sign shows that the current flows in the opposite direction of the current found when the capacitor is charging. **Figure 10.40**(b) shows an example of a plot of charge versus time and current versus time. A plot of the voltage difference across the capacitor and the voltage difference across the resistor as a function of time are shown in parts (c) and (d) of the figure. Note that the magnitudes of the charge, current, and voltage all decrease exponentially, approaching zero as time increases.

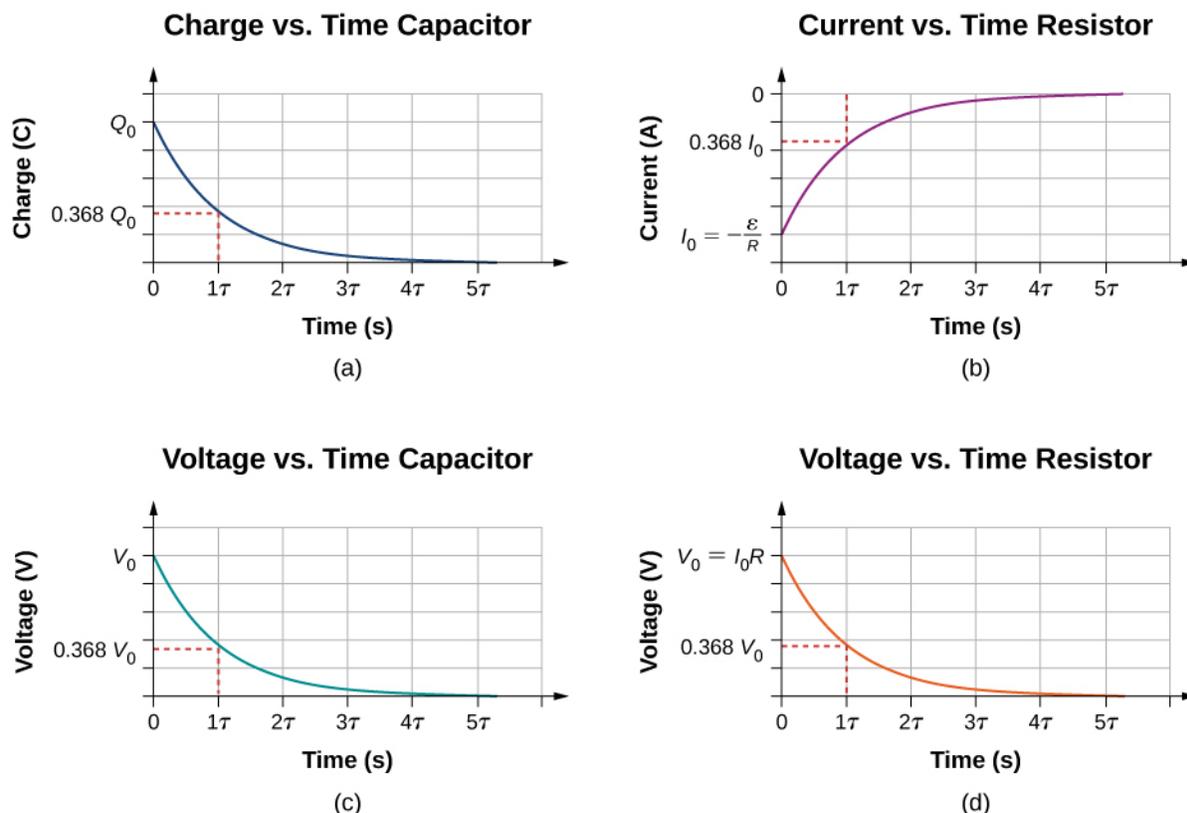


Figure 10.40 (a) Charge on the capacitor versus time as the capacitor discharges. (b) Current through the resistor versus time. (c) Voltage difference across the capacitor. (d) Voltage difference across the resistor.

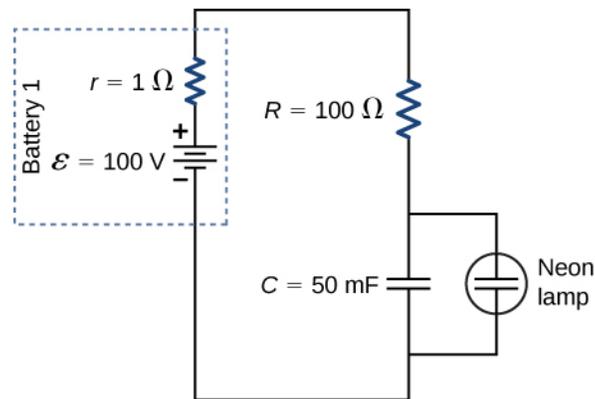
Now we can explain why the flash camera mentioned at the beginning of this section takes so much longer to charge than discharge: The resistance while charging is significantly greater than while discharging. The internal resistance of the battery accounts for most of the resistance while charging. As the battery ages, the increasing internal resistance makes the charging process even slower.

Example 10.8

The Relaxation Oscillator

One application of an RC circuit is the relaxation oscillator, as shown below. The relaxation oscillator consists of a voltage source, a resistor, a capacitor, and a neon lamp. The neon lamp acts like an open circuit (infinite resistance) until the potential difference across the neon lamp reaches a specific voltage. At that voltage, the lamp acts like a short circuit (zero resistance), and the capacitor discharges through the neon lamp and produces light. In the relaxation oscillator shown, the voltage source charges the capacitor until the voltage across the capacitor is 80 V. When this happens, the neon in the lamp breaks down and allows the capacitor to discharge through the lamp, producing a bright flash. After the capacitor fully discharges through the neon lamp, it begins to charge again, and the process repeats. Assuming that the time it takes the capacitor to discharge is negligible, what is the

time interval between flashes?



Strategy

The time period can be found from considering the equation $V_C(t) = \varepsilon(1 - e^{-t/\tau})$, where $\tau = (R + r)C$.

Solution

The neon lamp flashes when the voltage across the capacitor reaches 80 V. The RC time constant is equal to $\tau = (R + r)C = (101 \Omega)(50 \times 10^{-3} \text{ F}) = 5.05 \text{ s}$. We can solve the voltage equation for the time it takes the capacitor to reach 80 V:

$$\begin{aligned} V_C(t) &= \varepsilon(1 - e^{-t/\tau}), \\ e^{-t/\tau} &= 1 - \frac{V_C(t)}{\varepsilon}, \\ \ln(e^{-t/\tau}) &= \ln\left(1 - \frac{V_C(t)}{\varepsilon}\right), \\ t &= -\tau \ln\left(1 - \frac{V_C(t)}{\varepsilon}\right) = -5.05 \text{ s} \cdot \ln\left(1 - \frac{80 \text{ V}}{100 \text{ V}}\right) = 8.13 \text{ s}. \end{aligned}$$

Significance

One application of the relaxation oscillator is for controlling indicator lights that flash at a frequency determined by the values for R and C . In this example, the neon lamp will flash every 8.13 seconds, a frequency of $f = \frac{1}{T} = \frac{1}{8.13 \text{ s}} = 0.123 \text{ Hz}$. The relaxation oscillator has many other practical uses. It is often used in electronic circuits, where the neon lamp is replaced by a transistor or a device known as a tunnel diode. The description of the transistor and tunnel diode is beyond the scope of this chapter, but you can think of them as voltage controlled switches. They are normally open switches, but when the right voltage is applied, the switch closes and conducts. The “switch” can be used to turn on another circuit, turn on a light, or run a small motor. A relaxation oscillator can be used to make the turn signals of your car blink or your cell phone to vibrate.

RC circuits have many applications. They can be used effectively as timers for applications such as intermittent windshield wipers, pace makers, and strobe lights. Some models of intermittent windshield wipers use a variable resistor to adjust the interval between sweeps of the wiper. Increasing the resistance increases the RC time constant, which increases the time between the operation of the wipers.

Another application is the pacemaker. The heart rate is normally controlled by electrical signals, which cause the muscles of the heart to contract and pump blood. When the heart rhythm is abnormal (the heartbeat is too high or too low), pace makers can be used to correct this abnormality. Pacemakers have sensors that detect body motion and breathing to increase the heart rate during physical activities, thus meeting the increased need for blood and oxygen, and an RC timing circuit can be used to control the time between voltage signals to the heart.

Looking ahead to the study of ac circuits (**Alternating-Current Circuits**), ac voltages vary as sine functions with specific frequencies. Periodic variations in voltage, or electric signals, are often recorded by scientists. These voltage signals could come from music recorded by a microphone or atmospheric data collected by radar. Occasionally, these signals can contain

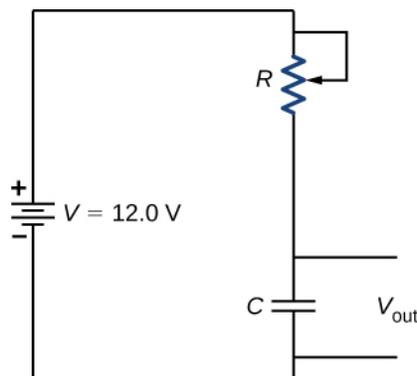
unwanted frequencies known as “noise.” RC filters can be used to filter out the unwanted frequencies.

In the study of electronics, a popular device known as a 555 timer provides timed voltage pulses. The time between pulses is controlled by an RC circuit. These are just a few of the countless applications of RC circuits.

Example 10.9

Intermittent Windshield Wipers

A relaxation oscillator is used to control a pair of windshield wipers. The relaxation oscillator consists of a 10.00-mF capacitor and a 10.00-k Ω variable resistor known as a rheostat. A knob connected to the variable resistor allows the resistance to be adjusted from 0.00 Ω to 10.00 k Ω . The output of the capacitor is used to control a voltage-controlled switch. The switch is normally open, but when the output voltage reaches 10.00 V, the switch closes, energizing an electric motor and discharging the capacitor. The motor causes the windshield wipers to sweep once across the windshield and the capacitor begins to charge again. To what resistance should the rheostat be adjusted for the period of the wiper blades be 10.00 seconds?



Strategy

The resistance considers the equation $V_{\text{out}}(t) = V(1 - e^{-t/\tau})$, where $\tau = RC$. The capacitance, output voltage, and voltage of the battery are given. We need to solve this equation for the resistance.

Solution

The output voltage will be 10.00 V and the voltage of the battery is 12.00 V. The capacitance is given as 10.00 mF. Solving for the resistance yields

$$\begin{aligned} V_{\text{out}}(t) &= V(1 - e^{-t/\tau}), \\ e^{-t/RC} &= 1 - \frac{V_{\text{out}}(t)}{V}, \\ \ln(e^{-t/RC}) &= \ln\left(1 - \frac{V_{\text{out}}(t)}{V}\right), \\ -\frac{t}{RC} &= \ln\left(1 - \frac{V_{\text{out}}(t)}{V}\right), \\ R &= \frac{-t}{C \ln\left(1 - \frac{V_{\text{out}}(t)}{V}\right)} = \frac{-10.00 \text{ s}}{10 \times 10^{-3} \text{ F} \ln\left(1 - \frac{10 \text{ V}}{12 \text{ V}}\right)} = 558.11 \Omega. \end{aligned}$$

Significance

Increasing the resistance increases the time delay between operations of the windshield wipers. When the resistance is zero, the windshield wipers run continuously. At the maximum resistance, the period of the operation of the wipers is:

$$t = -RC \ln\left(1 - \frac{V_{\text{out}}(t)}{V}\right) = -(10 \times 10^{-3} \text{ F})(10 \times 10^3 \Omega) \ln\left(1 - \frac{10 \text{ V}}{12 \text{ V}}\right) = 179.18 \text{ s} = 2.98 \text{ min}.$$

The RC circuit has thousands of uses and is a very important circuit to study. Not only can it be used to time circuits, it can also be used to filter out unwanted frequencies in a circuit and used in power supplies, like the one for your computer, to help turn ac voltage to dc voltage.

10.6 | Household Wiring and Electrical Safety

Learning Objectives

By the end of the section, you will be able to:

- List the basic concepts involved in house wiring
- Define the terms thermal hazard and shock hazard
- Describe the effects of electrical shock on human physiology and their relationship to the amount of current through the body
- Explain the function of fuses and circuit breakers

Electricity presents two known hazards: thermal and shock. A **thermal hazard** is one in which an excessive electric current causes undesired thermal effects, such as starting a fire in the wall of a house. A **shock hazard** occurs when an electric current passes through a person. Shocks range in severity from painful, but otherwise harmless, to heart-stopping lethality. In this section, we consider these hazards and the various factors affecting them in a quantitative manner. We also examine systems and devices for preventing electrical hazards.

Thermal Hazards

Electric power causes undesired heating effects whenever electric energy is converted into thermal energy at a rate faster than it can be safely dissipated. A classic example of this is the short circuit, a low-resistance path between terminals of a voltage source. An example of a short circuit is shown in **Figure 10.41**. A toaster is plugged into a common household electrical outlet. Insulation on wires leading to an appliance has worn through, allowing the two wires to come into contact, or “short.” As a result, thermal energy can quickly raise the temperature of surrounding materials, melting the insulation and perhaps causing a fire.

The circuit diagram shows a symbol that consists of a sine wave enclosed in a circle. This symbol represents an alternating current (ac) voltage source. In an ac voltage source, the voltage oscillates between a positive and negative maximum amplitude. Up to now, we have been considering direct current (dc) voltage sources, but many of the same concepts are applicable to ac circuits.

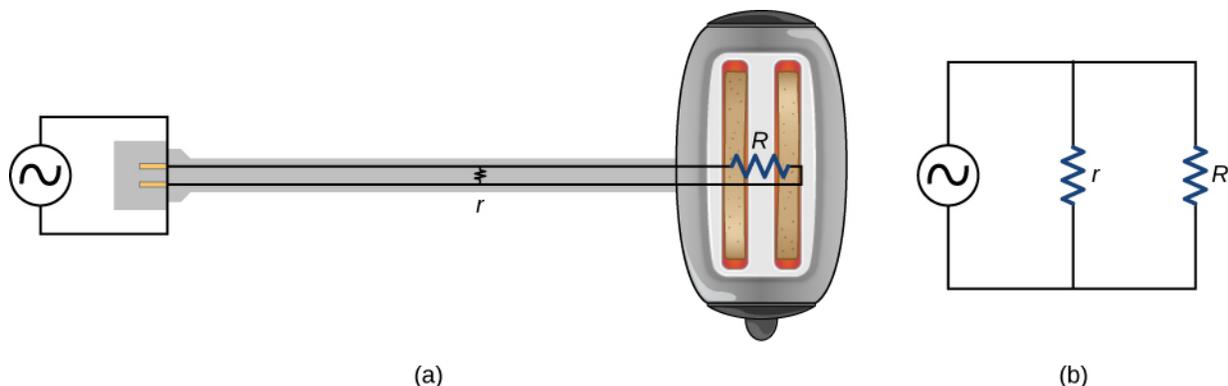


Figure 10.41 A short circuit is an undesired low-resistance path across a voltage source. (a) Worn insulation on the wires of a toaster allow them to come into contact with a low resistance r . Since $P = V^2/r$, thermal power is created so rapidly that the cord melts or burns. (b) A schematic of the short circuit.

Another serious thermal hazard occurs when wires supplying power to an appliance are overloaded. Electrical wires and appliances are often rated for the maximum current they can safely handle. The term “overloaded” refers to a condition where the current exceeds the rated maximum current. As current flows through a wire, the power dissipated in the supply wires is $P = I^2 R_W$, where R_W is the resistance of the wires and I is the current flowing through the wires. If either I or